MATH35012, Chapter 0: Before we begin ...

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Below is a (non-exhaustive) list of basic revision topics. Most of the things listed here should be obvious at this level; they will be used with little or no comment during the lectures.

1. Basic vector calculus. In particular you should be familiar with the definitions and notation:

$$\operatorname{grad} f \equiv \nabla f = (f_x, f_y, f_z)^T\,, \tag{1}$$
$$\operatorname{div} \underline{u} \equiv \nabla \cdot \underline{u} = u_x + v_y + w_z\,, \tag{2}$$
$$\operatorname{curl} \underline{u} \equiv \nabla \wedge \underline{u} \equiv \nabla \times \underline{u} = (w_y - v_z, u_z - w_x, v_x - u_y)^T\,, \tag{3}$$

where the subscripts denote differentiation with respect to the subscript variable. I don't expect you to have (m)any vector identities memorised, but the following are useful to know:

$$\nabla \wedge \nabla f = \underline{0}\,, \tag{4}$$
$$\nabla \cdot (\nabla \wedge \underline{u}) = 0\,, \tag{5}$$
$$\nabla \cdot \nabla f = \nabla^2 f\,. \tag{6}$$

2. The principle of superposition. Given any linear problem with a set of known solutions S_i, we can construct another solution which is the superposition $\sum_i W_i S_i$ for any set of constant weights W_i.

3. Fluid mechanics. I do not expect you to have done any fluid mechanics courses before. Hence we will assume without proof the equation for conservation of mass:

$$\rho_t + \nabla \cdot (\rho \underline{u}) = 0\,, \tag{7}$$

where ρ is the density of the fluid and $\underline{u}$ the velocity field, and Euler's equation:

$$\underline{u}_t + (\underline{u} \cdot \nabla)\underline{u} = -\frac{1}{\rho}\nabla p + \underline{G}\,, \tag{8}$$

where p is the pressure and $\underline{G}$ any body forces acting on the fluid (e.g. gravitational forces).

I do not expect you to derive (or memorise) these, only to use them in the wave-motion context that is discussed in the course.

Notation: The left-hand side of Euler's equation is often written using the 'material derivative' operator $D/Dt \equiv \partial_t + \underline{u} \cdot \nabla$. Our wave theory will be linear, so the nonlinear term will not make much of a contribution, but you are still expected to appreciate that the $(\underline{u} \cdot \nabla)\underline{u}$ notation is the equivalent of $u_j u_{i,j}$ in index notation, or that $(\underline{u} \cdot \nabla)$ is the differential operator $u\partial_x + v\partial_y + w\partial_z$.

4. Potential theory. If the curl of a vector field is zero everywhere ($\nabla \wedge \underline{u} = \underline{0}$) then there exists a scalar potential function ϕ such that $\underline{u} = \nabla\phi$. If we also know that $\nabla \cdot \underline{u} = 0$, then this leads to $\nabla \cdot \nabla\phi = 0$, which is Laplace's equation

$$\nabla^2\phi = 0\,. \tag{9}$$

I expect you to know the Cartesian form of Laplace's equation $\phi_{xx}+\phi_{yy}+\phi_{zz} = 0$. If we need its form for curvilinear coordinate systems, you will be given the corresponding definition or asked to derive it. Solutions to Laplace's equations are called 'harmonic functions'.

5. Separation of variables. You are expected to be familiar with the technique of separation of variables applied to linear PDEs. As part of the separation method you will also have to solve constant coefficient ODEs (see the core second semester, 1st year, calculus course). In particular, problems in the form

$$f''(x) + \lambda f'(x) + \mu^2 f(x) = 0\,, \tag{10}$$

should be trivial for you to solve at this level (ie. a harmonic oscillator with damping). Some familiarity with the ODEs arising from the separation of Laplace's equation in curvilinear coordinate systems will be helpful; but if you're not familiar with these, you will be after doing some of the later example sheets.

6. The harmonic equation. The case $\lambda = 0$ (no damping) in the above equation should be entirely trivial for you to write down a solution to. I also expect you to be able to switch between the multiple forms of the solution and realise when one form is more appropriate than another (as determined by the form of the boundary conditions). For example, if μ^2 is real and positive

$$\begin{aligned} f(x) &= a_1\cos(\mu x) + a_2\sin(\mu x)\,, && (11)\\ &= \bar{a}_1 e^{i\mu x} + \bar{a}_2 e^{-i\mu x}\,, && (12)\\ &= \hat{a}_1\cos(\mu x + \hat{a}_2)\,, && (13)\\ &= \tilde{a}_1\sin(\mu x + \tilde{a}_2)\,. && (14)\end{aligned}$$

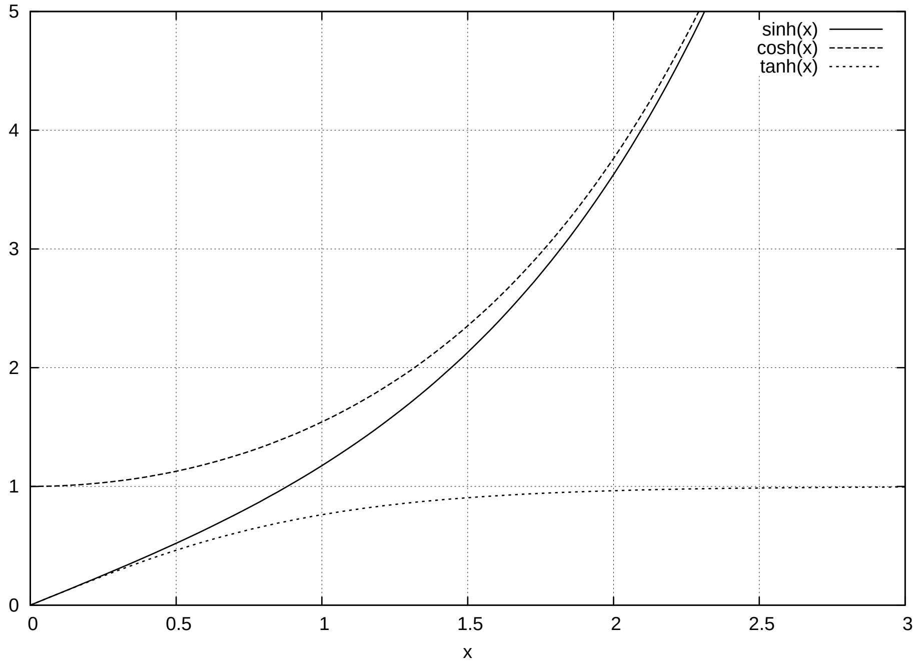
and if μ^2 is real and negative

$$\begin{aligned} f(x) &= b_1\cosh(\mu x) + b_2\sinh(\mu x)\,, && (15)\\ &= \bar{b}_1 e^{\mu x} + \bar{b}_2 e^{-\mu x}\,, && (16)\\ &= \hat{b}_1\cosh(\mu x + \hat{b}_2)\,, && (17)\\ &= \tilde{b}_1\sinh(\mu x + \tilde{b}_2)\,, && (18)\end{aligned}$$

for constants $a_1, b_1, \ldots$.

7. Trig identities. These will be useful in a handful of places in the course, but I don't expect you to have the full set memorised. However you should know $\sin^2\theta + \cos^2\theta = 1$, $\cosh^2\theta - \sinh^2\theta = 1$. You should also be able to express all circular and hyperbolic functions ($\sin, \cos, \tan, \sinh, \cosh, \tanh$) in terms of the exponential function.

8. Curve sketching. I expect you to be able to sketch basic functions, including all of the circular/hyperbolic functions (and know which are even/odd). Any curves that



you sketch must have appropriate labels and have the correct asymptotic behaviour where appropriate.

9. Fourier transforms. We will make use of Fourier transforms in a couple of places during the course. I will introduce them with no assumed knowledge, but if you have seen them before (e.g. in 'Applied complex analysis'), then you might find it helpful to review that material during the Easter break.
